universität wien

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Equilibrium Bitcoin Pricing

Public Lecture by OeNB Visiting Professor Bruno Biais

Standard money versus Cryptocurrency

Standard money: created by central and commercial banks

$\Rightarrow$ problem if institutions not trustworthy: inflation, expropriation (Zimbabwe, Venezuela, Turkey … China, financial crisis)

Cryptocurrency: created by internet network nodes, distributed

$\Rightarrow$ "allow online payments to be sent directly from one party to another without going through a financial institution" (Nakamoto, 2008) : Bitcoin

Can it work? Does bitcoin have value? Just fad, bubble bound to go to 0?

Distinguished warnings

"Money, is an indispensable social convention backed by an accountable institution within the State that enjoys public trust… Private digital tokens posing as currencies, such as bitcoin and other crypto-assets that have mushroomed of late, must not endanger this trust in the fundamental value and nature of money"

Agustin Carstens, BIS

"bitcoin is a pure bubble, an asset without intrinsic value — its price will fall to zero if trust vanishes"

Jean Tirole, TSE

Bumpy ride

Fundamental value underlying volatile price?

“You have to really stretch your imagination to infer what the intrinsic value of Bitcoin is. I haven’t been able to do it. Maybe somebody else can.” – Alan Greenspan, Bloomberg Interview, 4 December 2013

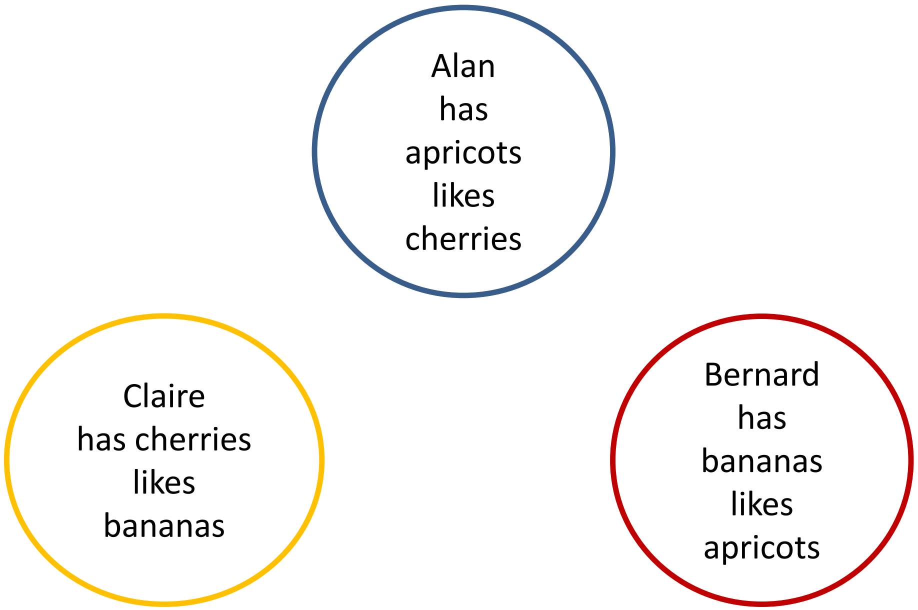
To address this question, standard, Overlapping Generations, Rational Expectations Equilibrium, model of money, adapted to bitcoin, confronted to data

What is money?

Something I am willing to accept today, in exchange for goods and services, because I anticipate the others will accept it tomorow, when I want to buy goods and services

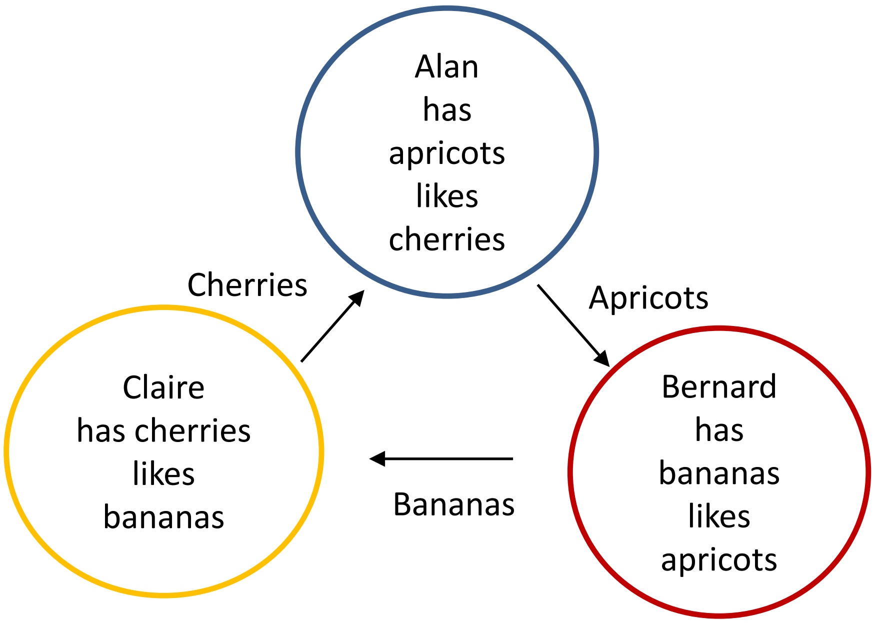
Money solves the non double coincidence of wants problem

Wicksell triangle



If all meet simultaneously in centralised clearing house (CCP)

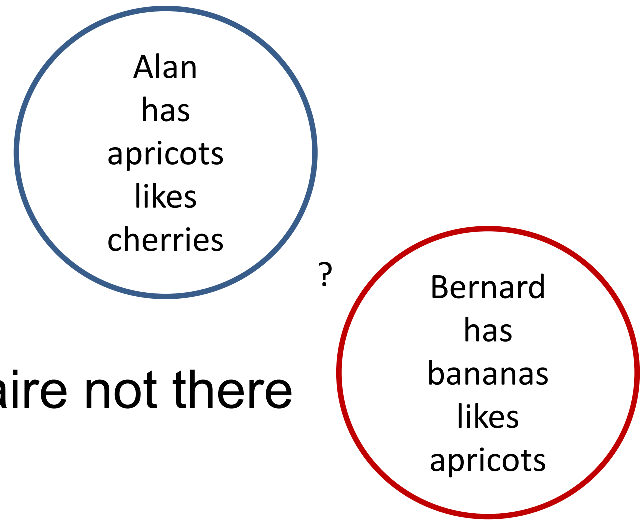
Use goods to pay for goods: payment in kind
Realise all gains from trade



Don’t need money. Use one good (e.g., apricots) as numéraire: price of apples & bananas in apricots

Non double coincidence of wants problems

not everybody simultaneously present on market



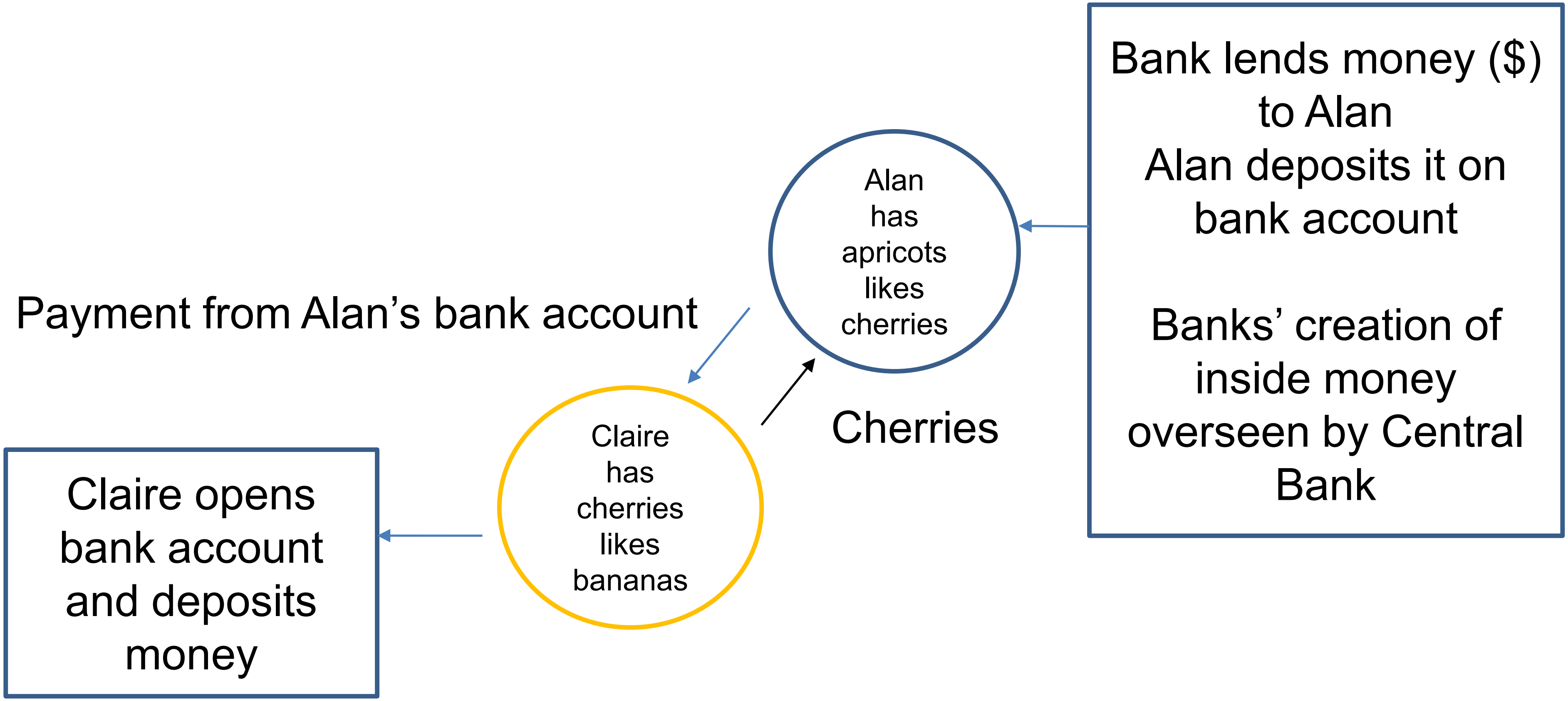
When Alan meets Bernard, Claire not there

Bernard likes Alan's apricots, but Alan not interested in Bernard's bananas

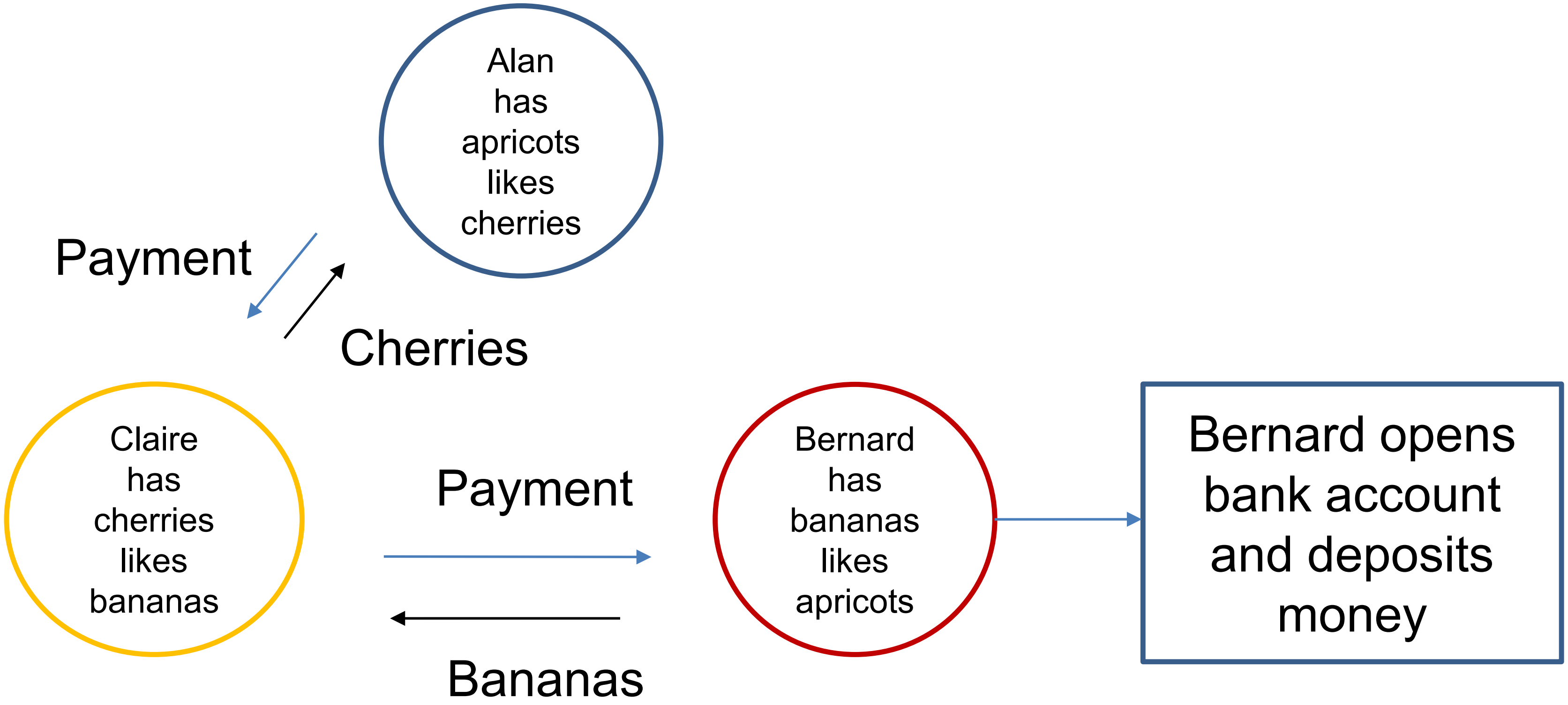
If bananas not storable, no trade: gains from trade not realised

Same thing when Bernard meets Claire, or Claire meets Alan

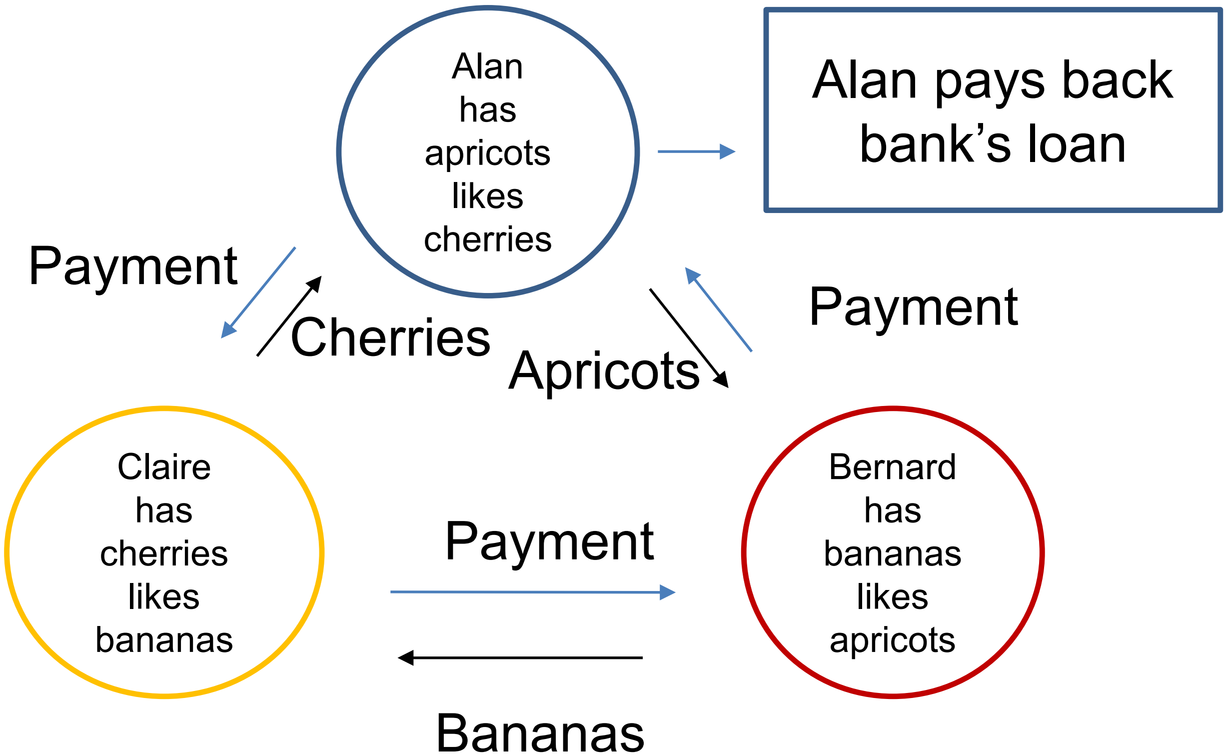
Monetary exchange with banks



Monetary exchange with banks



Monetary exchange with banks



With inside bank money, all gains from trades realised, but …

Need that, at each step, agents trust money to be accepted by others

When trust disappears, banks' payment system breaks down: collapse

=> during 2007-2008 crisis Fed eager to prevent bank payment system from collapsing (// Bernanke's research on1930s recession)

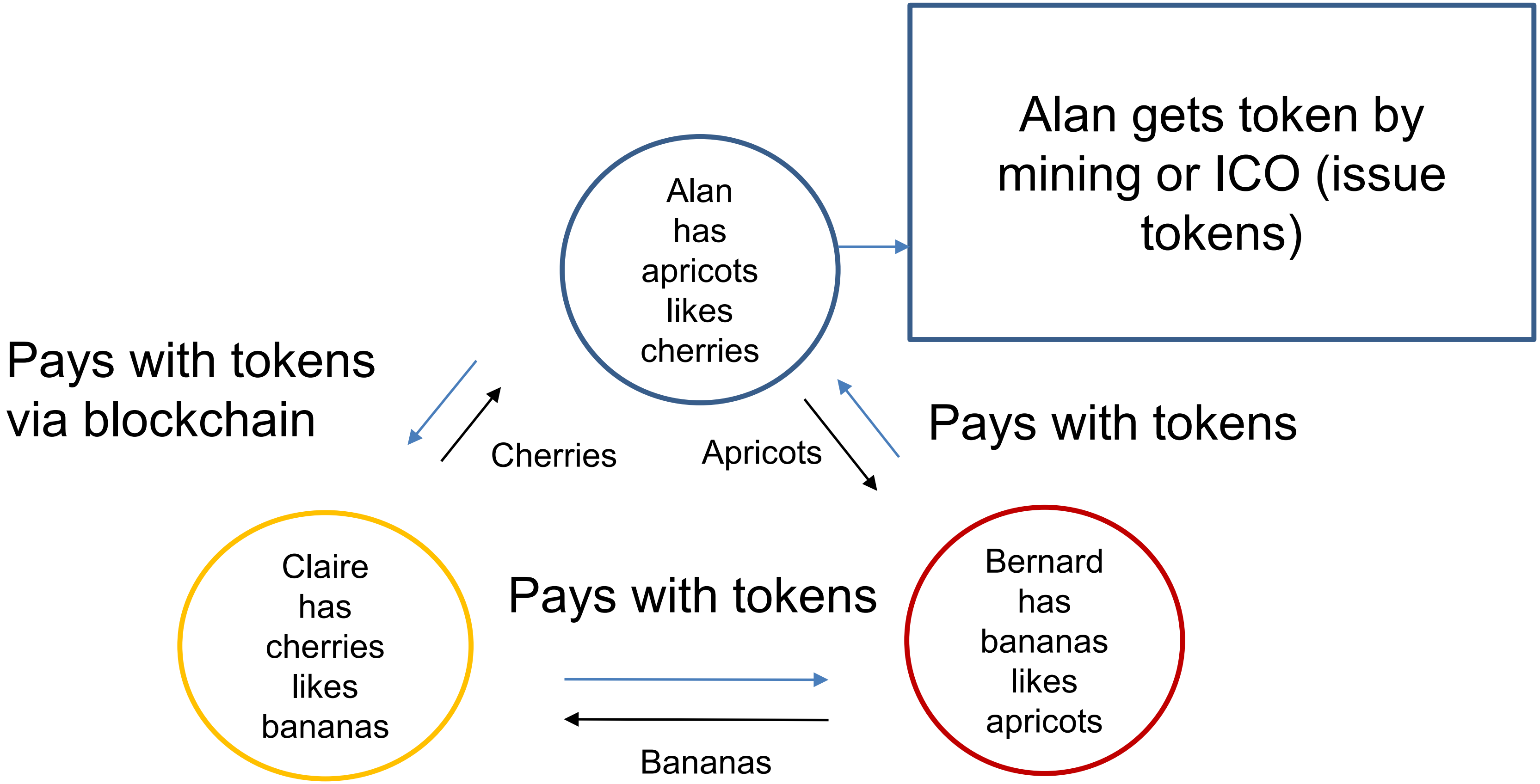
Cost of banks' payment system overseen by Central Bank:

If banks use antiquated technology or earn rents

If moral hazard: banks gamble, knowing Central bank will save them

If bad government expropriates or Central Bank inflates

Monetary exchange with Cryptocurrency



At each step, agents trust cryptocurrency (to be accepted by others) and blockchain… risk of collapse ?

With cryptocurrency all gains from trades realised, but …

If lose faith in acceptability of tokens, collapse η

Exchanges where crypto traded for dollars,yens or euros can be hacked: *h*

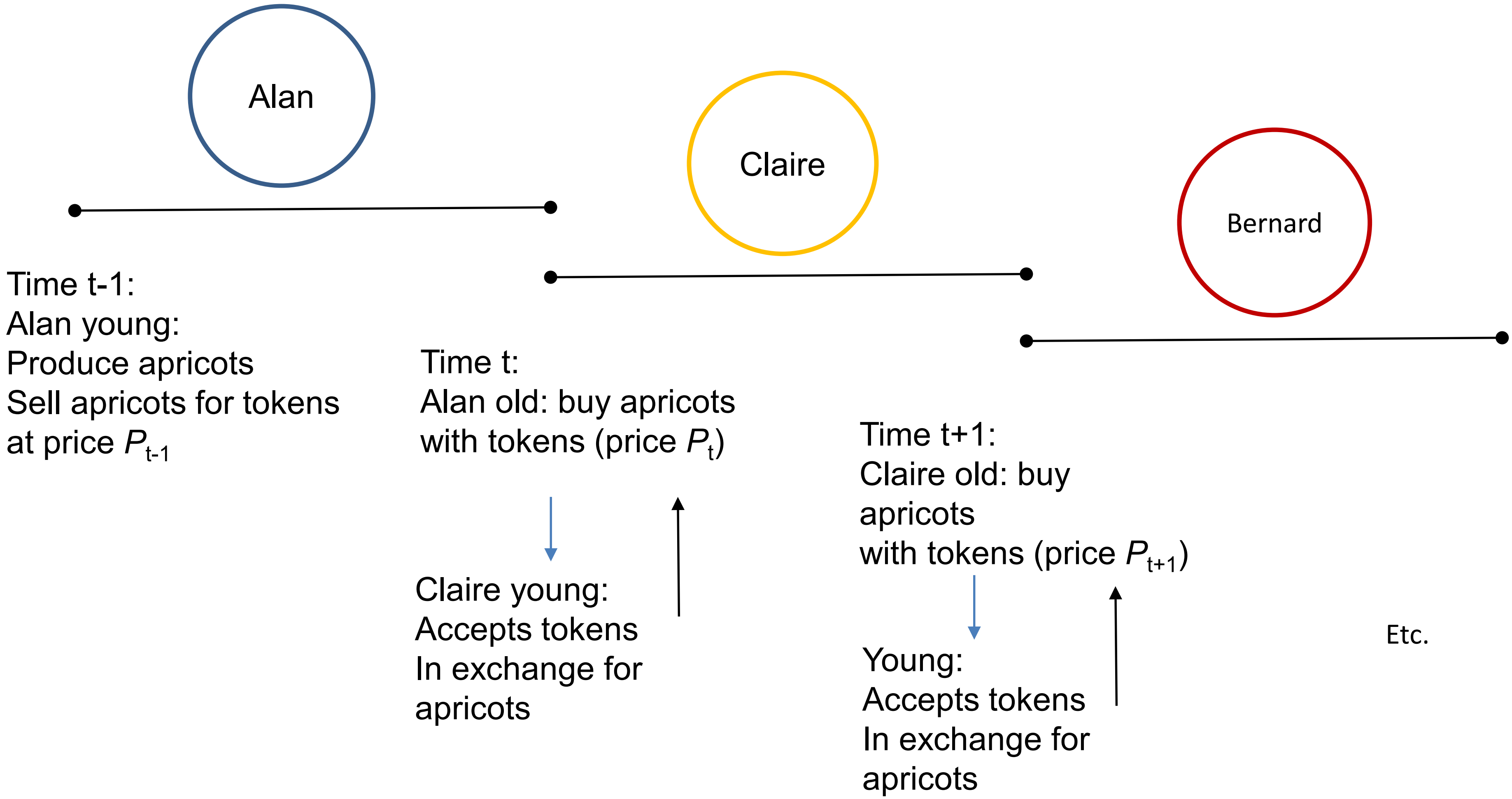
Trading crypto involves transactions fees (miners or exchanges): φ

Yet, you can do things with cryptos that you can't do with banking system: transfer wealth abroad (China), avoid expropriation or inflation (Zimbabwe, Venezuela): θ

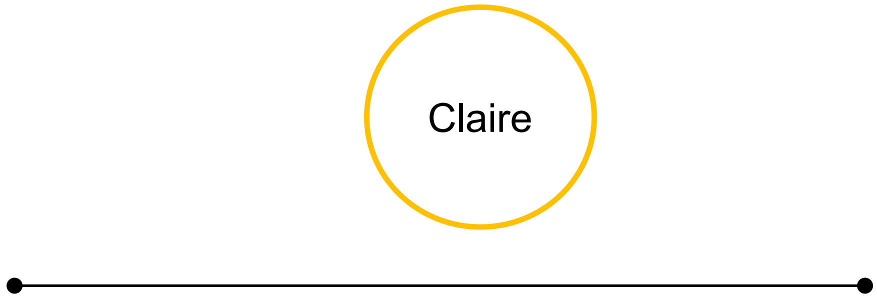
And, usefulness of crypto-tokens increases as more sellers of goods and services accept them as payment (Expedia, Dell, etc.): θ

To examine these issues, non-double coincidence of wants model:
+ econometric estimation of *h*, φ, η and θ

Simple way to model non double coincidence of wants: OLG



REE: agents optimise, rational expectations on $\{P_t\}$, market clears



Young at t:
Accepts tokens at price P_t
(transaction fee φ)

In exchange for apricots

Old at t+1:
Use (unhacked fraction $1-h$ of) tokens
which she sells at price P_{t+1}

To buy apricots (highly valued if θ large)

Purchasing power of token large if their price
increased, large return $\rho = (P_{t+1}/P_t) - 1$

Budget constraints in OLG model

Young Claire consumes endowment, saves, buys dollar, and crypto incurring transactions cost (φ_t)

$$c_t^y = e_t^y - s_t - q_t p_t - \hat{q}_t \hat{p}_t - \varphi_t(q_t) p_t.$$

p_t = price of cryto in consumption goods

Old Claire consumes endowment, savings, dollars and crypto hoarded: fraction h_{t+1} of crypto stolen, but transactional benefits (θ_{t+1})

$$c_{t+1}^o = e_{t+1}^o + s_t(1 + r_t) + (1 - h_{t+1})(1 + \theta_{t+1}) q_t p_{t+1} + \hat{q}_t \hat{p}_{t+1}$$

Similar to Kareken Wallace 1981 – Garratt Wallace 2018 except that we have hack risk (h_{t+1}), transactions costs (φ_t) & benefits (θ_{t+1}), risk-aversion

Equilibrium

$$\max_{q_t, s_t, \hat{q}_t} u(c_t^y) + \beta E_t u(c_{t+1}^o)$$

s.t., $$c_t^y = e_t^y - s_t - q_t p_t - \hat{q}_t \hat{p}_t - \varphi_t(q_t) p_t.$$

$$c_{t+1}^o = e_{t+1}^o + s_t(1 + r_t) + (1 - h_{t+1})(1 + \theta_{t+1}) q_t p_{t+1} + \hat{q}_t \hat{p}_{t+1}$$

F.O.C w.r.t q_t + market clearing: $q_t = X_t$

$$p_t = \beta E_t \left[\frac{u'(c_{t+1}^o)}{u'(c_t^y)} (1 - h_{t+1}) \frac{(1 + \theta_{t+1})}{(1 + \varphi_t'(X_t))} p_{t+1} \right]$$

F.O.C w.r.t s_t

$$\beta = \frac{1}{1 + r_t} \frac{u'(c_t^y)}{E_t \left[u'(c_{t+1}^o) \right]}.$$

Transactions costs and benefits notation

Transactions benefits of using bitcoin: not expropriated/taxed/constrained by government, direct internet access, …

$$1 + \mathcal{T}_{t+1} = \frac{1 + \theta_{t+1}}{1 + \varphi_t'(X_t)}$$

Cost of buying bitcoin with dollars: transactions costs charged by exchanges, miners' fees, …

Necessary condition for equilibrium price (Euler equation)

Combining two first order conditions and using notation $\mathscr{T}$

$$p_t = \frac{1}{1+r_t} E_t \left(\frac{u'(c^o_{t+1})}{E_t\left[u'(c^o_{t+1})\right]} (1 - h_{t+1}) \left(p_{t+1} + \mathcal{T}_{t+1} p_{t+1}\right) \right)$$

discount — risk neutral proba — hack risk — resale price — transactional net benefit

Similar to Tirole 1985 (OLG model of money) except that:

- randomness
- hack risk, transactions costs & benefits

Comparison with stock price

$$p_t = \frac{1}{1+r_t} E_t \left(\frac{u'(c^o_{t+1})}{E_t\left[u'(c^o_{t+1})\right]} (1 - h_{t+1}) \left(p_{t+1} + \mathcal{T}_{t+1} p_{t+1}\right) \right)$$

discount | risk neutral proba | hack risk | resale price | transactional net benefit

stock not exposed to hack risk

dividend d_{t+1} instead of transactional benefit $T_{t+1}\, p_{t+1}$

dividends = fundamental of stock

transactional benefits = fundamental of currency

Iterating

$$p_t = E_t\left[\frac{1-h_{t+1}}{1+r_t}\frac{u'(c^o_{t+1})}{E_t\left[u'(c^o_{t+1})\right]}(p_{t+1}+\mathcal{T}_{t+1}p_{t+1})\right]$$

$$p_{t+1} = E_{t+1}\left[\frac{1-h_{t+2}}{1+r_{t+1}}\frac{u'(c^o_{t+2})}{E_{t+1}\left[u'(c^o_{t+2})\right]}(p_{t+2}+\mathcal{T}_{t+2}p_{t+2})\right]$$

$$p_t = E_t\left[\begin{array}{c} \frac{1-h_{t+1}}{1+r_t}\frac{u'(c^o_{t+1})}{E_t\left[u'(c^o_{t+1})\right]}\mathcal{T}_{t+1}p_{t+1} + \frac{1-h_{t+1}}{1+r_t}\frac{u'(c^o_{t+1})}{E_t\left[u'(c^o_{t+1})\right]}\frac{1-h_{t+2}}{1+r_{t+1}}\frac{u'(c^o_{t+2})}{E_t\left[u'(c^o_{t+2})\right]}\mathcal{T}_{t+2}p_{t+2} \\ + \frac{1-h_{t+1}}{1+r_t}\frac{u'(c^o_{t+1})}{E_t\left[u'(c^o_{t+1})\right]}\frac{1-h_{t+2}}{1+r_{t+1}}\frac{u'(c^o_{t+2})}{E_t\left[u'(c^o_{t+2})\right]}p_{t+2} \end{array}\right]$$

Present value of transactional benefits

$$p_t = E_t\left(\sum_{k=1}^{K}\left(\prod_{j=1}^{k}\frac{1-h_{t+j}}{1+r_{t+j-1}}\frac{u'(c^o_{t+j})}{E_t\left[u'(c^o_{t+j})\right]}\mathcal{T}_{t+j}p_{t+j}\right)+\left(\prod_{j=1}^{K}\frac{1-h_{t+j}}{1+r_{t+j-1}}\frac{u'(c^o_{t+j})}{E_t\left[u'(c^o_{t+j})\right]}\right)p_{t+K}\right) \quad (8)$$

Price = present value stream of transactional benefits

As beliefs fluctuate about future transactional benefits (acceptability in future, future ease of exchange against goods, services and other currencies) and future price

Current price also fluctuates

Multiplicative structure (unlike stocks)

$$p_t = \frac{1}{1+r_t} E_t \left(\frac{u'(c^o_{t+1})}{E_t\left[u'(c^o_{t+1})\right]} (1 - h_{t+1}) \left(p_{t+1} + \mathcal{T}_{t+1} p_{t+1}\right) \right)$$

Stocks: dividend d_{t+1} not multiplied by price

$\Rightarrow$ dividend anchors price (with something outside price)

Currency: transactional benefit $T_{t+1}\ p_{t+1}$ multiplied by price

$\Rightarrow$ current price depends on expectation of future price, no anchor

$\Rightarrow$ multiple equilibria (Kareken Wallace 1981 ``indeterminacy result'')
Price = 0 is one of many possible equilibria

For simplicity assume risk neutrality

Liu and Tsyvinski (2018) : correlation bitcoin consumption, production, income economically and statistically insignificant

$$p_t = \frac{1}{1+r_t} E_t \left(\frac{u'(c^o_{t+1})}{E_t\left[u'(c^o_{t+1})\right]} (1 - h_{t+1}) \left(p_{t+1} + \mathcal{T}_{t+1} p_{t+1}\right) \right)$$

simplifies to

$$p_t = \frac{1}{1+r_t} E_t \left((1 - h_{t+1}) \left(1 + \mathcal{T}_{t+1}\right) p_{t+1} \right)$$

Consequences of multiplicative structure: Exogenous volatility

$$p_t = \frac{1}{1+r_t} E_t \left(\left(1 - h_{t+1}\right) \left(1 + \mathcal{T}_{t+1}\right) p_{t+1} \right)$$

In a given equilibrium: exogenous random shocks (sunspots)

Take above equation, suppose it holds until t, it still holds if prices after t multiplied by exogenous iid random variable with mean 1

$$p^*_s = p_s, \text{ for all } s \leq t$$

$$p^*_s = p_s\,(\eta_{t+1} \ldots \eta_s), \text{ for all } s > t$$

$\Rightarrow$ volatility, unrelated to fundamentals

Because no real anchor, only beliefs: Shiller 1981 critique does not apply (in REE currency can move lots more than fundamentals)

Cryptocurrency price in $

F.O.C w.r.t $ holdings

$$\hat{p}_t = \frac{1}{1 + r_t} E_t \left(\hat{p}_{t+1} \right)$$

Assumption A2 *Inflation in the central bank currency between time t and time $t+1$ is known at time t.*

$$\hat{p}_t = \frac{\hat{p}_{t+1}}{1 + r_t}$$

Divide crypto price by $ price

$$\frac{p_t}{\hat{p}_t} = \frac{\frac{1}{1+r_t} E_t \left[\left(1 - h_{t+1} \right) \left(1 + \mathcal{T}_{t+1} \right) p_{t+1} \right]}{\frac{\hat{p}_{t+1}}{1+r_t}}.$$

Interest rate cancels

$$\frac{p_t}{\hat{p}_t} = E_t \left[\left(1 - h_{t+1} \right) \left(1 + \mathcal{T}_{t+1} \right) \frac{p_{t+1}}{\hat{p}_{t+1}} \right]$$

Cryptocurrency dollar returns

$$\rho_{t+1} = \frac{\frac{p_{t+1}}{\hat{p}_{t+1}}}{\frac{p_t}{\hat{p}_t}} - 1$$

1 dollar today
worth 1 dollar in 1 week

$$E_t \left[(1 - h_{t+1}) \frac{1 + \theta_{t+1}}{1 + \varphi_t'(X_t)} (1 + \rho_{t+1}) \right] - 1 = 0.$$

If use dollar to buy btc:

fraction *h* of btc can be stolen (hacked)

transaction costs φ to trade bitcoin (e.g., fees)

can use bitcoin to trade differently than with dollar $\rightarrow$ value θ

$\Rightarrow$ Moment condition used to test model and estimate parameters = costs (φ) and benefits (θ) of holding btc

First order approximation

$$E_t\left[\rho_{t+1}\right] \approx \varphi_t'(X_t) + E_t(h_{t+1}) - E_t(\theta_{t+1}).$$

Equilibrium required expected return

costs

benefits

Goal of econometric analysis

Test rational expectations equilibrium pricing relation

using observed returns (ρ_{t+1}) and hacks (h_{t+1})

and observable variables to proxy for θ_{t+1} and φ_t

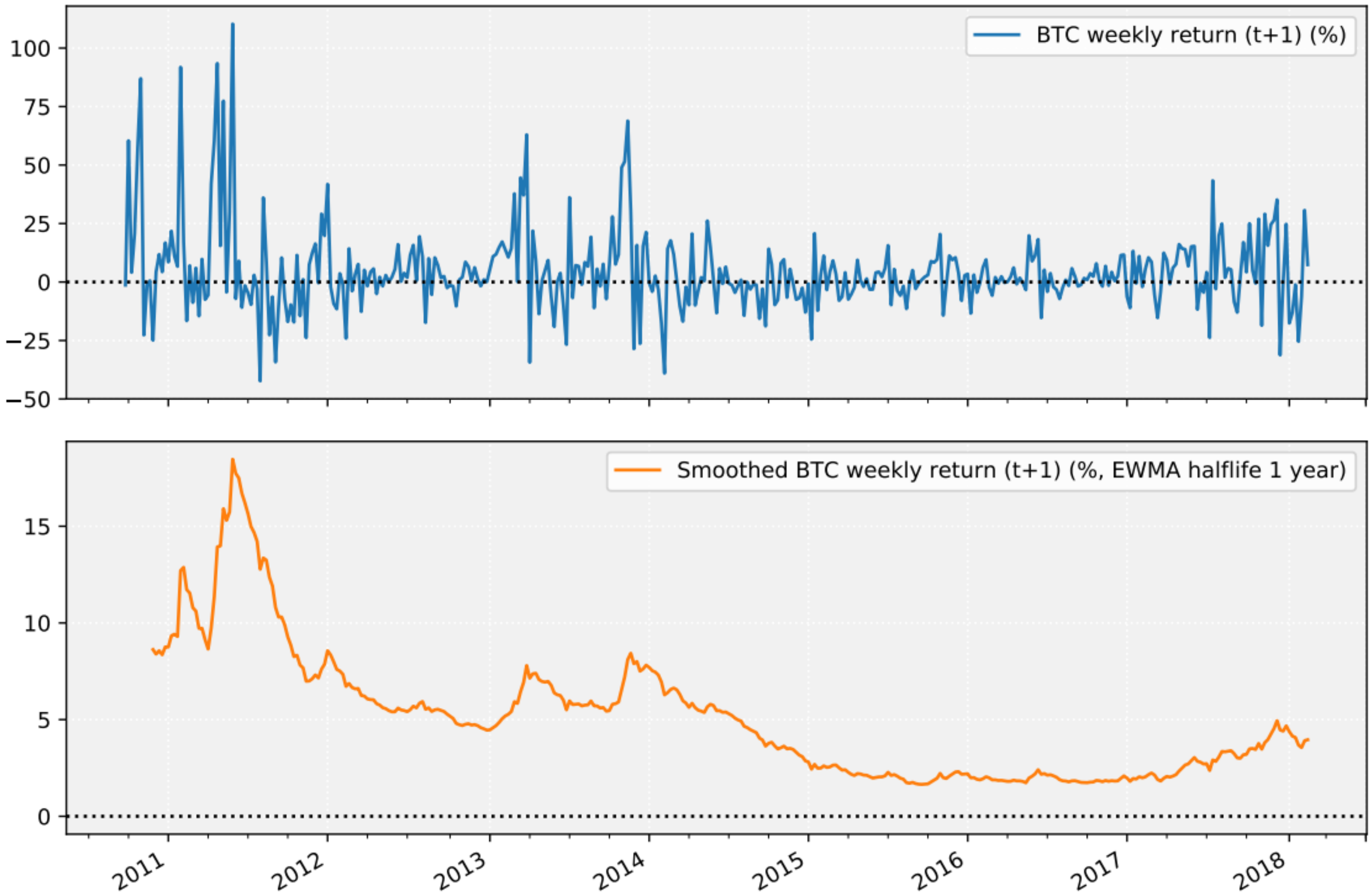
$\Rightarrow$ estimate fundamental value and costs of bitcoin

(relying on Generalised Method of Moments, Hansen 1982)

$\Rightarrow$ Is REE rejected ?

$\Rightarrow$ Is hypothesis that fundamental value significantly positive rejected?

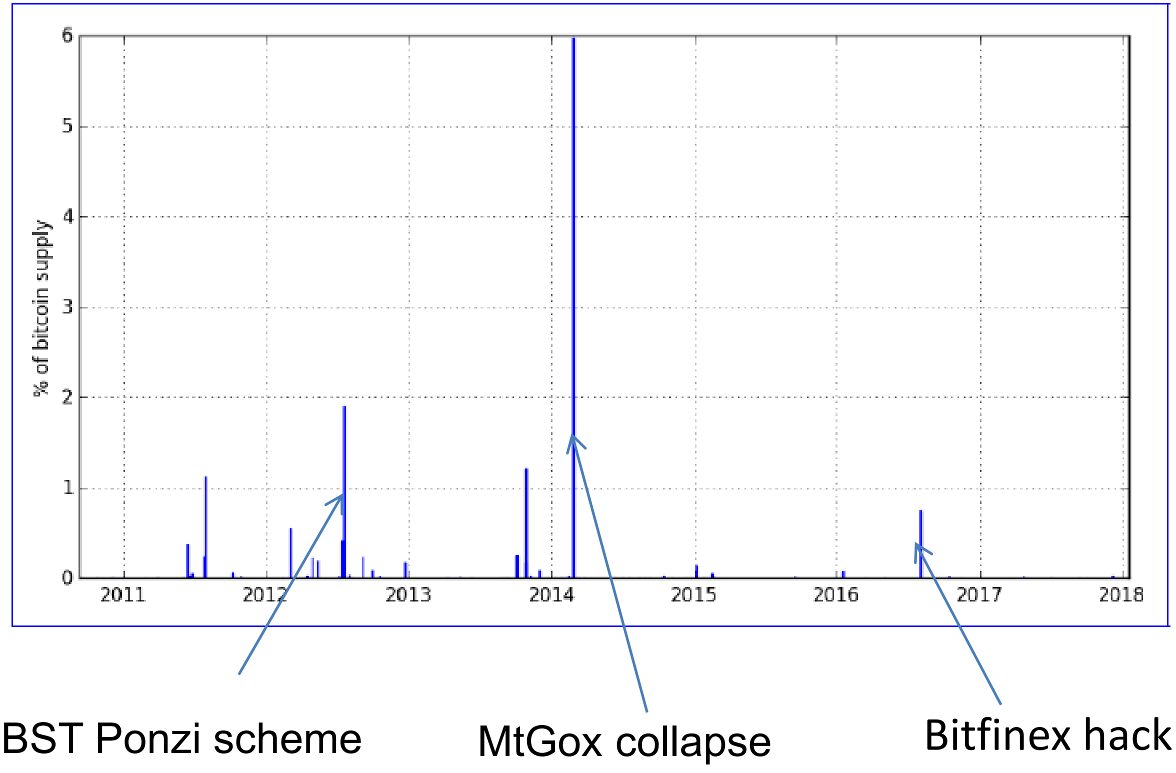
Time series of weekly btc returns "variable to be explained"



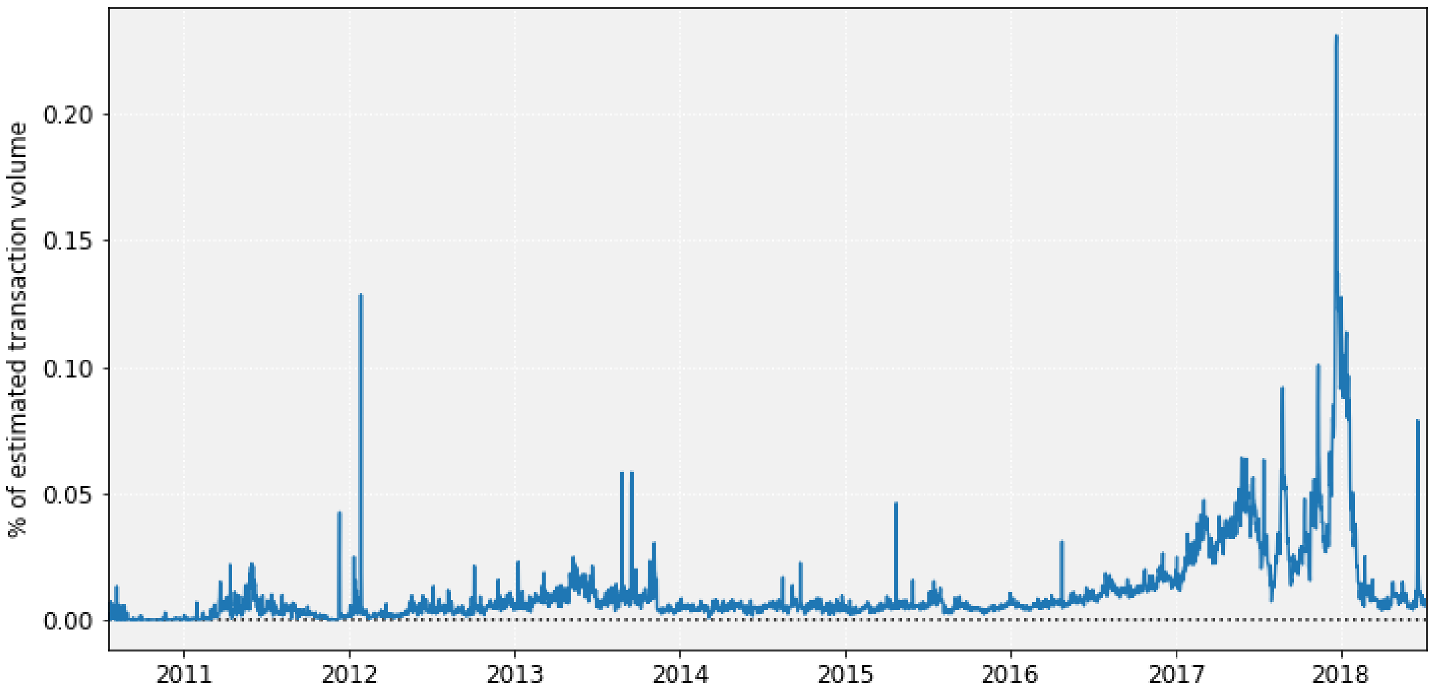
418 obs to be confronted to hacks, transactions costs and benefits

Browsing the net, construct time series of bitcoin thefts/hacks to serve as estimate of *h*

on average 0.04% of btc supply stolen per week
(so hack risk can explain only .04 percentage points of btc required returns)



Download blockchain, construct time series of transaction fees, requested by miners for including transactions in blocks, to proxy for φ



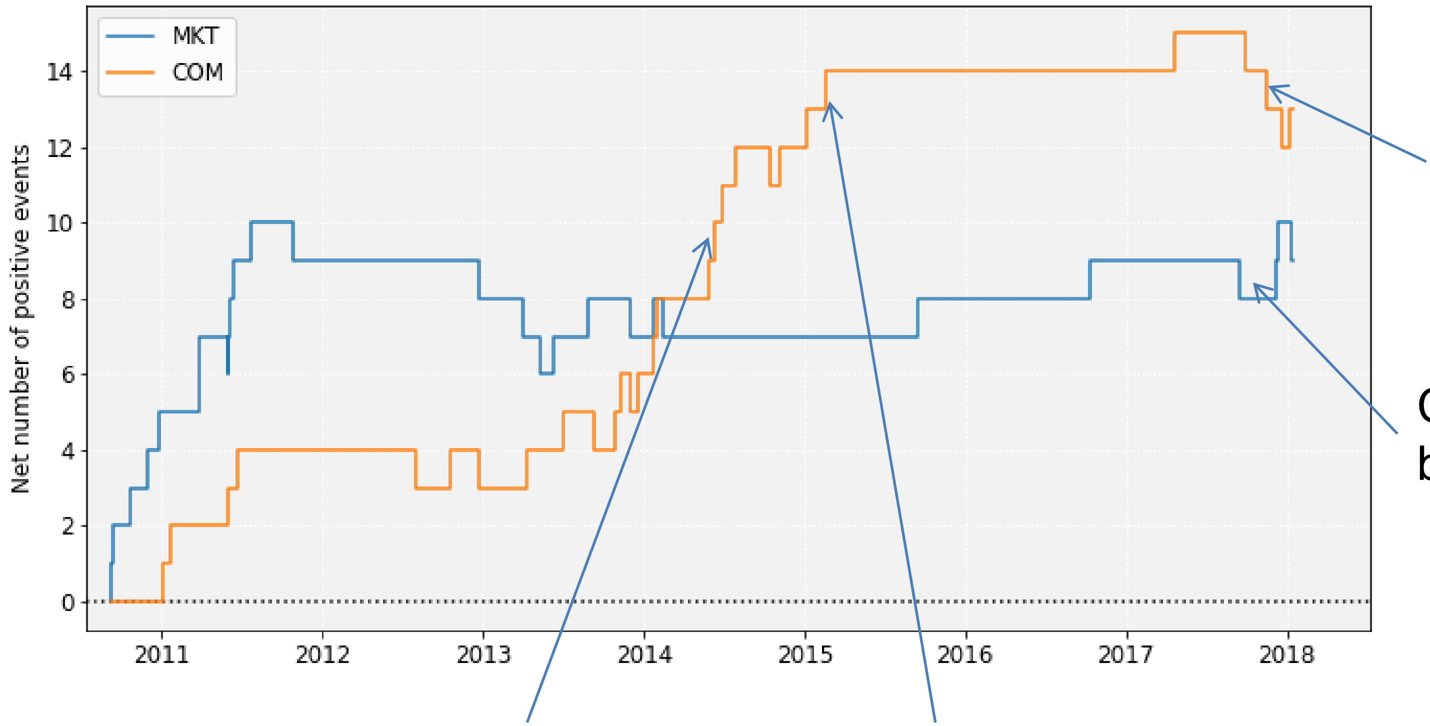
Larger transactions fees towards end of 2017 (rise in btc attracts many investors, raise transaction volume, congest network)

Otherwise fees rather low

Browsing net, find events affecting:

- ability to trade bitcoin for dollars, to also help proxy for φ
- ease to trade bitcoin for goods and services, to proxy for θ

Coding each positive as +1 and negative as -1, index of ease to trade btc



Dell no longer accepts btc

China bans btc

Expedia accepts btc   Paypal accepts btc   btc future in CME

Ability to exchange btc for $ improved early (MKT), ease to exchange btc for goods/services improved later (COM)

GMM

Impose moment condition

$$E_t\left[(1-h_{t+1})\frac{1+\theta_{t+1}}{1+\varphi_t'(X_t)}(1+\rho_{t+1})\right]-1=0.$$

with θ affine in COM and φ affine in MKT and transaction fees:

$$\theta = \alpha_1 \text{ COM}_{t+1}$$

$$\phi'_t = \beta_0 + \beta_1 \text{ fee}_t + \beta_2 \text{ COM}_{t+1}$$

Parameters to be estimated: α, β

Instruments: year dummies, COM, MKT, transaction fees, lagged btc return, …

Table 1: GMM estimates of model parameters

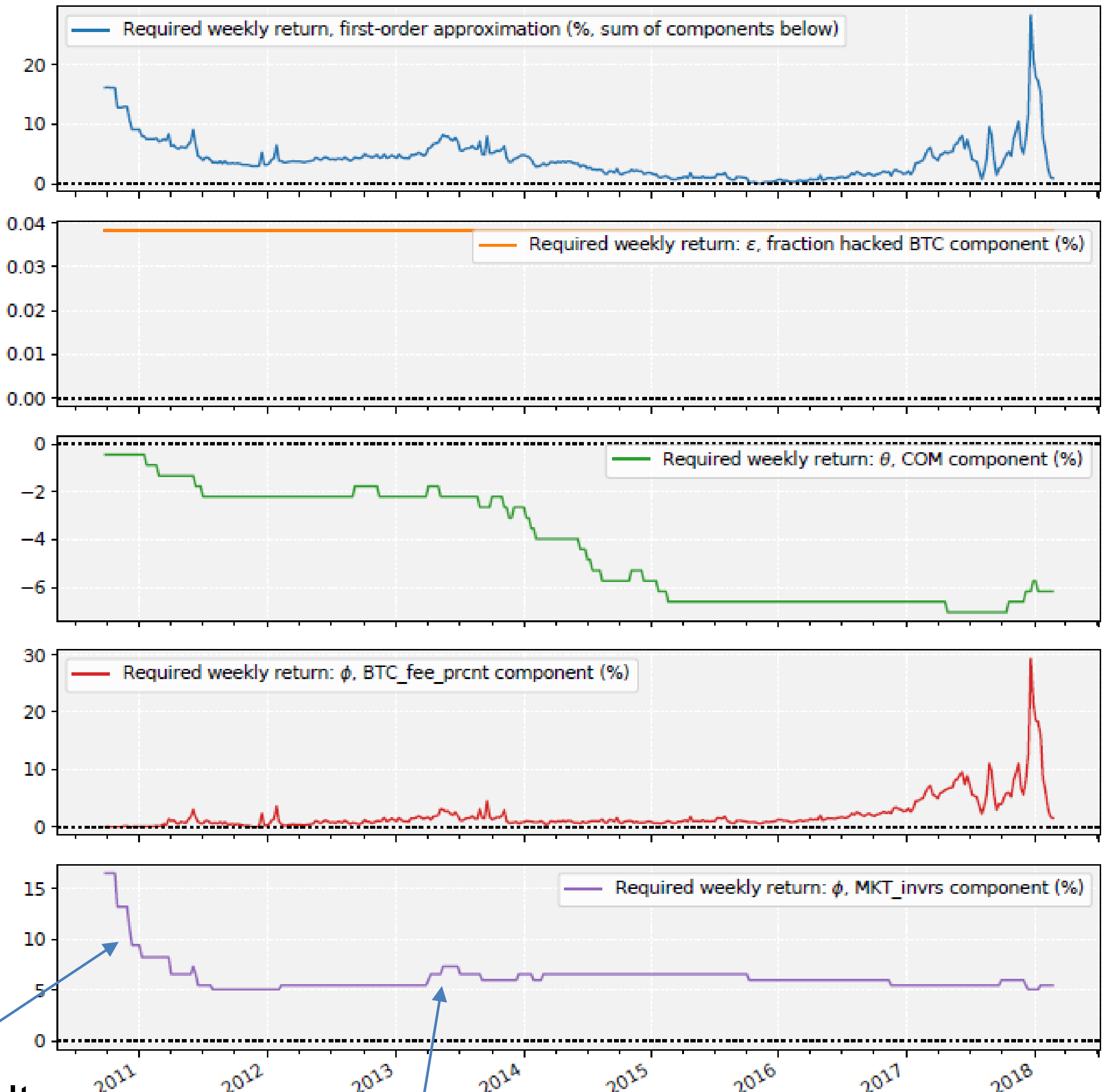
This table presents the GMM estimates of the model parameters. t-values are in parentheses and statistical significance is indicated by one, two, or three stars that correspond to a 10%, 5%, or 1% level, respectively.

Parameter	Variable	Model (1)	Model (2)
α_1	COM_{t+1}	0.00364*** (2.65)	0.00441*** (3.31)
β_0	*Intercept*	−0.0583 (−1.06)	
β_1	$BTC\_fee\_prcnt_t$	1.60* (1.94)	1.86** (2.45)
β_2	MKT_invrs_t	1.20** (2.06)	0.661*** (3.79)

// theory, required E(return) significantly decreasing in transaction benefits (COM: θ) and increasing in unease to exchange btc for dollar (MKT_invrs φ)

Transactions fees have right sign, but not really significant (maybe too small)

$$E_t\left[\rho_{t+1}\right] \approx \varphi_t'(X_t) + E_t(h_{t+1}) - E_t(\theta_{t+1}).$$



Initially very difficult to exchange btc for goods and services

Then became easier, but still difficult → Commands important fraction of required return

Changes in fundamentals vs non-fundamental noise

standard deviation observed btc weekly return: 17.9%

standard deviation estimated btc required weekly return: 3.43%

implied R2: (0.034^2)/(0.179^2) = 3.67%

Fundamentals explain part of btc fluctuations

But large fraction of fluctuations reflects exogenous noise (sunspots, changes in beliefs, …)

Tentative conclusion

Theory:

fundamental of currency (including btc) = transaction services

multiplicative structure => exogenous volatility in REE (no Shiller bounds)

Econometric analysis:

preliminary…

fundamentals (e.g., ease to use btc to buy goods and services) seem to explain some of btc fluctuations, but large part reflects exogenous noise